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POPULATION MODEL WITH CROSS-DIFFUSION WITH DOUBLE NONLINEARITY

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ABSTRACT

Considering the parabolic system of the two quasi linear equations, of the reaction-diffusion problem, of Kolmogorov-Fisher type was for biological population. The two-dimensional case and localization of wave solutions of the systems of reaction – diffusion, with double nonlinearity was done. Cross-diffusion means that, spatial movement of a single object, which is described in one of the variables, is due to the diffusion of another object, described by another variable. We considered a spatial analogue of Volterra-Lotka competition system, with nonlinear power dependence of the diffusion coefficient on the density of the population.

KEYWORDS: Reaction-Diffusion, Diffusion with Double Nonlinearity, Spatial Move & Density of the Population

1. INTRODUCTION

Let's consider the following system of two equations in the two-dimensional case:

$$\begin{aligned}\frac{\partial u_1}{\partial t} &= f(u_1,u_2) + D_{11}\frac{\partial^2 u_1}{\partial x_1^{\,2}} + D_{12}\frac{\partial^2 u_1}{\partial x_2^{\,2}} + h_{11}\frac{\partial}{\partial x_1}\left(Q_1(u_1,u_2)\frac{\partial u_2}{\partial x_1}\right) + h_{12}\frac{\partial}{\partial x_2}\left(Q_1(u_1,u_2)\frac{\partial u_2}{\partial x_2}\right),\\ \frac{\partial u_2}{\partial t} &= g(u_1,u_2) + D_{21}\frac{\partial^2 u_2}{\partial x_1^{\,2}} + D_{22}\frac{\partial^2 u_2}{\partial x_2^{\,2}} + h_{21}\frac{\partial}{\partial x_1}\left(Q_2(u_1,u_2)\frac{\partial u_1}{\partial x_1}\right) + h_{22}\frac{\partial}{\partial x_2}\left(Q_2(u_1,u_2)\frac{\partial u_1}{\partial x_2}\right).\end{aligned} \tag{1}$$

At $h_{11} = h_{12} = h_{21} = h_{22} = 0$ mathematical model (1) is a system of the reaction-diffusion type with diffusion coefficients $D_{11} \geq 0,\ D_{12} \geq 0, D_{21} \geq 0,\ D_{22} \geq 0$ (at least one $D_{ij} \neq 0$). In the case where at least one of the coefficients (a sign can be anything), the system (1) is the cross-diffusion. Linear cross-diffusion corresponds to $Q_{ij}(u,v) = const$ for $i, j = 1,2$ I =1, 2; nonlinear cross-diffusion $Q_{ij}(u,v) \neq const$ at least one of i and j.

Cross-diffusion means that spatial movement of a single object, which is described in one of the variables is due to the diffusion of another object, described by another variable. At the population level, the simplest example-a parasite (the first object) within the "host" (the second object) moves due to the diffusion of the host. The term "self-diffusion" (diffusion, direct diffusion, ordinary diffusion) involves the movement of individuals at the expense of the diffusion flow from areas of high concentration especially in the region of their low concentration. The term "cross-diffusion" refers to the movement/flow of individuals of one species/ substances due to the presence of the gradient other individuals/ substances. The magnitude of the cross-diffusion coefficient can be positive, negative or equal to zero. The positive coefficient of cross-diffusion indicates that the movement of individuals occurs in the direction of the low concentration of other individuals in the direction of high concentrations of other types of individuals/ substances. In nature, systems with cross diffusion are quite common and play a significant role, especially in biophysical and biomedical systems.

Equation (1) is a generalization of the simple diffusion model for the logistic model of population growth [1-16] of Malthus type ($f_1(u_1,u_2)=u_1$, $f_1(u_1,u_2)=u_2$, $f_2(u_1,u_2)=u_1$, $f_2(u_1,u_2)=u_2$), Ferhulst type ($f_1(u_1,u_2)=u_1(1-u_2)$, $f_1(u_1,u_2)=u_2(1-u_1)$, $f_2(u_1,u_2)=u_1(1-u_2)$, $f_2(u_1,u_2)=u_2(1-u_1)$), and Olli type ($f_1(u_1,u_2)=u_1(1-u_2^{\beta_1})$, $f_1(u_1,u_2)=u_2(1-u_1^{\beta_2})$, $f_2(u_1,u_2)=u_1(1-u_2^{\beta_1})$, $f_2(u_1,u_2)=u_2(1-u_1^{\beta_2})$, $\beta_1>1,\ \beta_2>1$) for the case of double nonlinear diffusion. In the case when $\beta_1\geq 1,\ \beta_2\geq 1$, it can be regarded as the equation of nonlinear filtration, thermal conductivity, while the impact source and the absorption capacity of which is equal respectively $u_1, -u_2^{\beta_1}$, $u_2, -u_1^{\beta_2}$.

Consider the spatial analogue of Volterra-Lotka system with non-linear power dependence of the diffusion coefficient on the density of the population. In the case of the simplest Volterra competitive interactions between populations can be constructed numerically, and in some cases analytically heterogeneous in space solutions [19].

2. LOCALIZATION OF WAVE SOLUTIONS OF SYSTEMS OF REACTION - DIFFUSION WITH DOUBLE NONLINEARITY

Let's consider Q= {(t, x): 0< t <∞, x∈ R^2} a parabolic system of two quasilinear equations of the reaction-diffusion problem of biological population of Kolmogorov-Fisher type

$$\begin{cases} \dfrac{\partial u_1}{\partial t}=\dfrac{\partial}{\partial x_1}\left(D_{11}u_2^{m_1-1}\left|\dfrac{\partial u_1}{\partial x_1}\right|^{p-2}\dfrac{\partial u_1}{\partial x_1}\right)+\dfrac{\partial}{\partial x_2}\left(D_{12}u_2^{m_1-1}\left|\dfrac{\partial u_1}{\partial x_2}\right|^{p-2}\dfrac{\partial u_1}{\partial x_2}\right)+l_1(t)\dfrac{\partial u_1}{\partial x_1}+l_2(t)\dfrac{\partial u_1}{\partial x_2}+k_1(t)u_1\left(1-u_2^{\beta_1}\right), \\ \dfrac{\partial u_2}{\partial t}=\dfrac{\partial}{\partial x_1}\left(D_{21}u_1^{m_2-1}\left|\dfrac{\partial u_2}{\partial x_1}\right|^{p-2}\dfrac{\partial u_2}{\partial x_1}\right)+\dfrac{\partial}{\partial x_2}\left(D_{2}u_1^{m_2-1}\left|\dfrac{\partial u_2}{\partial x_2}\right|^{p-2}\dfrac{\partial u_2}{\partial x_2}\right)+l_1(t)\dfrac{\partial u_2}{\partial x_1}+l_2(t)\dfrac{\partial u_2}{\partial x_2}+k_2(t)u_2\left(1-u_1^{\beta_2}\right), \end{cases} \qquad (2)$$

$$u_1\big|_{t=0}=u_{10}(x),\ u_2\big|_{t=0}=u_{20}(x),$$

which describes the process of biological populations in a nonlinear two-component environment, the diffusion coefficient of which is equal to $D_{11}u_2^{m_1-1}\left|\frac{\partial u_1}{\partial x_1}\right|^{p-2}$, $D_{12}u_2^{m_1-1}\left|\frac{\partial u_1}{\partial x_2}\right|^{p-2}$, $D_{21}u_1^{m_2-1}\left|\frac{\partial u_2}{\partial x_1}\right|^{p-2}$, $D_{22}u_1^{m_2-1}\left|\frac{\partial u_2}{\partial x_2}\right|^{p-2}$ and convective transfer at a speed of $l_i(t)$, where $m_1, m_2, p, \beta_1, \beta_2$ - positive real numbers, $u_1=u_1(t,x_1,x_2)\geq 0$, $u_2=u_2(t,x_1,x_2)\geq 0$ - desired solution.

The Cauchy problem and boundary value problem for system (1) in one-dimensional and multidimensional cases investigated by many authors [15-21].

The aim of this work is to study the qualitative properties of the solution of the problem (2) based on the self-similar analysis and its numerical solution using the methods of modern computer technologies, research on methods of linearization to the convergence of the iterative process with further visualization. The estimates of solutions and the resulting free boundary; that gives an opportunity to choose the appropriate initial approximation [15], for each value of the numerical parameters.

It is known that nonlinear equations have a wave solution in the form of diffusion waves. Under the wave refers to the self-similar solution of equation (2) of the form

$$u(t,x) = f(\xi),\ \xi = ct \pm x$$

$$u(t,x) = u(t,x_1,x_2),\quad x = \sqrt{(x_1)^2 + (x_2)^2}\ ,$$

Where the constant c is the wave speed

Let's build self-similar system of equations for (2) is simpler to study the system of equations.

Self-similar system of equations we will construct by the method of nonlinear splitting [15].

Substitution in (2)

$$u_1(t,x_1,x_2) = e^{-\int_0^t k_1(\zeta)d\zeta} v_1(\tau(t),\eta_1,\eta_2)\,,\ \eta_1 = x_1 - \int_0^t l_1(\zeta)d\zeta\,,$$

$$u_2(t,x_1,x_2) = e^{-\int_0^t k_2(\zeta)d\zeta} v_2(\tau(t),\eta_1,\eta_2)\,,\ \eta_2 = x_2 - \int_0^t l_2(\zeta)d\zeta\,,$$

Result (2) can be written as:

$$\begin{cases} \dfrac{\partial v_1}{\partial \tau} = \dfrac{\partial}{\partial \eta_1}\left(D_{11} v_2^{m_1-1}\left|\dfrac{\partial v_1}{\partial \eta_1}\right|^{p-2}\dfrac{\partial v_1}{\partial \eta_1}\right) + \dfrac{\partial}{\partial \eta_2}\left(D_{12} v_2^{m_1-1}\left|\dfrac{\partial v_1}{\partial \eta_2}\right|^{p-2}\dfrac{\partial v_1}{\partial \eta_2}\right) - k_1(t)e^{[(2-p)k_1+(\beta_1-m_1+1)k_2]t} v_1 v_2^{\beta_1}, \\ \dfrac{\partial v_2}{\partial \tau} = \dfrac{\partial}{\partial \eta_1}\left(D_{21} v_1^{m_2-1}\left|\dfrac{\partial v_2}{\partial \eta_1}\right|^{p-2}\dfrac{\partial v_2}{\partial \eta_1}\right) + \dfrac{\partial}{\partial \eta_2}\left(D_{22} v_1^{m_2-1}\left|\dfrac{\partial v_2}{\partial \eta_2}\right|^{p-2}\dfrac{\partial v_2}{\partial \eta_2}\right) - k_2(t)e^{[(\beta_2-m_2+1)k_1+(2-p)k_2]t} v_1^{\beta_2} v_2, \end{cases} \tag{3}$$

$$v_1\big|_{t=0} = v_{10}(\eta_1,\eta_2)\,,\ v_2\big|_{t=0} = v_{20}(\eta_1,\eta_2)\,.$$

If $k_1(p-(m_1+1)) = k_2(p-(m_2+1))$, by choosing $\tau(t) = \dfrac{e^{[(m_1-1)k_2+(p-2)k_1]t}}{(m_1-1)k_2+(p-2)k_1} = \dfrac{e^{[(m_2-1)k_1+(p-2)k_2]t}}{(m_2-1)k_1+(p-2)k_2}$, get the following system of equations:

$$\begin{cases} \dfrac{\partial v_1}{\partial \tau} = \dfrac{\partial}{\partial \eta_1}\left(D_{11} v_2^{m_1-1}\left|\dfrac{\partial v_1}{\partial \eta_1}\right|^{p-2}\dfrac{\partial v_1}{\partial \eta_1}\right) + \dfrac{\partial}{\partial \eta_2}\left(D_{12} v_2^{m_1-1}\left|\dfrac{\partial v_1}{\partial \eta_2}\right|^{p-2}\dfrac{\partial v_1}{\partial \eta_2}\right) - a_1(t)\tau^{b_1} v_1 v_2^{\beta_1}, \\ \dfrac{\partial v_2}{\partial \tau} = \dfrac{\partial}{\partial \eta_1}\left(D_{21} v_1^{m_2-1}\left|\dfrac{\partial v_2}{\partial \eta_1}\right|^{p-2}\dfrac{\partial v_2}{\partial \eta_1}\right) + \dfrac{\partial}{\partial \eta_2}\left(D_{22} v_1^{m_2-1}\left|\dfrac{\partial v_2}{\partial \eta_2}\right|^{p-2}\dfrac{\partial v_2}{\partial \eta_2}\right) - a_2(t)\tau^{b_2} v_1^{\beta_2} v_2, \end{cases} \tag{4}$$

Where $a_1 = k_1((p-2)k_1 + (m_1-1)k_2)^{b_1}$, $b_1 = \dfrac{(2-p)k_1 + (\beta_1 - m_1 + 1)k_2}{(p-2)k_1 + (m_1-1)k_2}$,

$$a_2 = k_2((m_2-1)k_1 + (p-2)k_2)^{b_2}\,,\ b_2 = \frac{(\beta_2 - m_2 + 1)k_1 + (2-p)k_2}{(m_2-1)k_1 + (p-2)k_2}.$$

If $b_i = 0$,and $a_i(t) = const$, $i = 1,2$, the system has the form:

$$\begin{cases} \dfrac{\partial v_1}{\partial \tau} = \dfrac{\partial}{\partial \eta_1}\left(D_{11} v_2^{m_1-1} \left| \dfrac{\partial v_1}{\partial \eta_1} \right|^{p-2} \dfrac{\partial v_1}{\partial \eta_1} \right) + \dfrac{\partial}{\partial \eta_2}\left(D_{12} v_2^{m_1-1} \left| \dfrac{\partial v_1}{\partial \eta_2} \right|^{p-2} \dfrac{\partial v_1}{\partial \eta_2} \right) - a_1 v_1 v_2^{\beta_1}, \\ \dfrac{\partial v_2}{\partial \tau} = \dfrac{\partial}{\partial \eta_1}\left(D_{21} v_1^{m_2-1} \left| \dfrac{\partial v_2}{\partial \eta_1} \right|^{p-2} \dfrac{\partial v_2}{\partial \eta_1} \right) + \dfrac{\partial}{\partial \eta_2}\left(D_{22} v_1^{m_2-1} \left| \dfrac{\partial v_2}{\partial \eta_2} \right|^{p-2} \dfrac{\partial v_2}{\partial \eta_2} \right) - a_2 v_1^{\beta_2} v_2. \end{cases}$$

The Cauchy problem for a system (4) in the case, when $b_1 = b_2 = 0$ studied in [16, 19] and proved the existence of a wave of global solutions and blow-up solutions.

Below we describe one way of obtaining self-similar system for the system of equations (4). It consists of the following. We find first the solution of a system of ordinary differential equations

$$\begin{cases} \dfrac{d\bar{v}_1}{d\tau} = -a_1 \bar{v}_1 \bar{v}_2^{\beta_1}, \\ \dfrac{d\bar{v}_2}{d\tau} = -a_2 \bar{v}_1^{\beta_2} \bar{v}_2, \end{cases}$$

In the form

$$\bar{v}_1(\tau) = c_1(\tau + T_0)^{-\gamma_1},\ \bar{v}_2(\tau) = c_2(\tau + T_0)^{-\gamma_2},\ T_0 > 0,$$

where

$$c_1 = 1,\ \gamma_1 = \frac{1}{\beta_2},\ c_2 = 1,\ \gamma_2 = \frac{1}{\beta_1}.$$

Then the solution of system (3)-(4) is sought in the form

$$\begin{aligned} v_1(t,\eta_1,\eta_2) &= \bar{v}_1(t) w_1(\tau,\eta_1,\eta_2), \\ v_2(t,\eta_1,\eta_2) &= \bar{v}_2(t) w_2(\tau,\eta_1,\eta_2), \end{aligned} \tag{5}$$

and $\tau = \tau(t)$ is chosen

$$\tau_1(\tau) = \int_0^\tau \bar{v}_1^{(p-2)}(t) \bar{v}_2^{(m_1-1)}(t)\,dt = \begin{cases} \dfrac{1}{1-[\gamma_1(p-2)+\gamma_2(m_1-1)]}(T+\tau)^{1-[\gamma_1(p-2)+\gamma_2(m_1-1)]}, & if\ \ 1-[\gamma_1(p-2)+\gamma_2(m_1-1)] \neq 0, \\ \ln(T+\tau), & if\ \ 1-[\gamma_1(p-2)+\gamma_2(m_1-1)] = 0, \\ (T+\tau), & if\ \ p=2\ \ u\ \ m_1 = 1, \end{cases}$$

if $\gamma_1(p-2) + \gamma_2(m_1 - 1) = \gamma_2(p-2) + \gamma_1(m_2 - 1)$.

Then, for $w_i(\tau, x),\ \ i = 1,2$. get the system of equations.

$$\begin{cases} \dfrac{\partial w_1}{\partial \tau} = \dfrac{\partial}{\partial \eta_1}\left(D_{11} w_2^{m_1-1} \left| \dfrac{\partial w_1}{\partial \eta_1} \right|^{p-2} \dfrac{\partial w_1}{\partial \eta_1} \right) + \dfrac{\partial}{\partial \eta_2}\left(D_{12} w_2^{m_1-1} \left| \dfrac{\partial w_1}{\partial \eta_2} \right|^{p-2} \dfrac{\partial w_1}{\partial \eta_2} \right) + \psi_1 (w_1 w_2^{\beta_1} - w_1) \\ \dfrac{\partial w_2}{\partial \tau} = \dfrac{\partial}{\partial \eta_1}\left(D_{21} w_1^{m_2-1} \left| \dfrac{\partial w_2}{\partial \eta_1} \right|^{p-2} \dfrac{\partial w_2}{\partial \eta_1} \right) + \dfrac{\partial}{\partial \eta_2}\left(D_{22} w_1^{m_2-1} \left| \dfrac{\partial w_2}{\partial \eta_2} \right|^{p-2} \dfrac{\partial w_2}{\partial \eta_2} \right) + \psi_2 (w_2 w_1^{\beta_2} - w_2) \end{cases} \tag{6}$$

Where,

$$\psi_1 = \begin{cases} \dfrac{1}{(1-[\gamma_1(p-2)+\gamma_2(m_1-1)])\tau}, & if \;\; 1-[\gamma_1(p-2)+\gamma_2(m_1-1)>0, \\ \gamma_1 c_1^{-(1-[\gamma_1(p-2)+\gamma_2(m_1-1)])}, & if \;\; 1-[\gamma_1(p-2)+\gamma_2(m_1-1)=0, \end{cases} \qquad (7)$$

$$\psi_2 = \begin{cases} \dfrac{1}{(1-[\gamma_2(p-2)+\gamma_1(m_2-1)])\tau}, & если \;\; 1-[\gamma_2(p-2)+\gamma_1(m_2-1)]>0, \\ \gamma_2 c_1^{-(1-[\gamma_2(p-2)+\gamma_1(m_2-1)])}, & если \;\; 1-[\gamma_2(p-2)+\gamma_1(m_2-1)]=0. \end{cases}$$

The performance of the system (2) to (5) suggests that, when $\tau \to \infty$, $\psi_i \to 0$ and

$$\begin{cases} \dfrac{\partial w_1}{\partial \tau} = \dfrac{\partial}{\partial \eta_1}\left(D_{11} w_2^{m_1-1}\left|\dfrac{\partial w_1}{\partial \eta_1}\right|^{p-2} \dfrac{\partial w_1}{\partial \eta_1}\right) + \dfrac{\partial}{\partial \eta_2}\left(D_{12} w_2^{m_1-1}\left|\dfrac{\partial w_1}{\partial \eta_2}\right|^{p-2} \dfrac{\partial w_1}{\partial \eta_2}\right) \\ \dfrac{\partial w_2}{\partial \tau} = \dfrac{\partial}{\partial \eta_1}\left(D_{21} w_1^{m_2-1}\left|\dfrac{\partial w_2}{\partial \eta_1}\right|^{p-2} \dfrac{\partial w_2}{\partial \eta_1}\right) + \dfrac{\partial}{\partial \eta_2}\left(D_{22} w_1^{m_2-1}\left|\dfrac{\partial w_2}{\partial \eta_2}\right|^{p-2} \dfrac{\partial w_2}{\partial \eta_2}\right) \end{cases} \qquad (8)$$

Therefore, the solution of system (1) with conditions (5) tends to the solution of the system (8).

If, $1-[\gamma_1(p-2)+\gamma_2(m_1-1)=0$, wave solution of system (6) has the form

$$w_i(\tau(t),\eta_1,\eta_2) = y_i(\xi),\; \xi = c\tau \pm \eta, \quad \eta = \sqrt{(\eta_1)^2+(\eta_2)^2} \;\; i=1,2,$$

Where, C is the wave velocity, and given that the equation for $w_i(\tau,\eta_1,\eta_2)$ without the younger members always has a self-similar solution in the case $1-[\gamma_1(p-2)+\gamma_2(m_1-1)\neq 0$ receive system

$$\begin{cases} \dfrac{d}{d\xi}(y_2^{m_1-1}\left|\dfrac{dy_1}{d\xi}\right|^{p-2} \dfrac{dy_1}{d\xi}) + c\dfrac{dy_1}{d\xi} + \psi_1(y_1 - y_1\; y_2^{\beta_1}) = 0, \\ \dfrac{d}{d\xi}(y_1^{m_2-1}\left|\dfrac{dy_2}{d\xi}\right|^{p-2} \dfrac{dy_2}{d\xi}) + c\dfrac{dy_2}{d\xi} + \psi_2(y_2 - y_2\; y_1^{\beta_2}) = 0. \end{cases}$$

After integrating (8) we get the system of nonlinear differential equations of the first order

$$\begin{cases} y_2^{m_1-1}\left|\dfrac{dy_1}{d\xi}\right|^{p-2} \dfrac{dy_1}{d\xi} + cy_1 = 0, \\ y_1^{m_2-1}\left|\dfrac{dy_2}{d\xi}\right|^{p-2} \dfrac{dy_2}{d\xi} + cy_2 = 0. \end{cases} \qquad (9)$$

The system (9) has the approximate solution

$$\bar{y}_1 = A(a-\xi)^{\gamma_1}, \ \bar{y}_2 = B(a-\xi)^{\gamma_2},$$

Where,

$$\gamma_1 = \frac{(p-1)(p-(m_1+1))}{(p-2)^2-(m_1-1)(m_2-1)},$$

$$\gamma_2 = \frac{(p-1)(p-(m_2+1))}{(p-2)^2-(m_1-1)(m_2-1)}.$$

And the coefficients A and B are determined from the solution of a system of nonlinear algebraic equations

$$(\gamma_1)^{p-1} A^{p-1} B^{m_1-1} = c,$$

$$(\gamma_2)^{p-1} A^{m_2-1} B^{p-1} = c.$$

Then from the expression

$$u_1(t,x_1,x_2) = e^{-\int_0^t k_1(\zeta)d\zeta} v_1(\tau(t),\eta_1,\eta_2),$$

$$u_2(t,x_1,x_2) = e^{-\int_0^t k_2(\zeta)d\zeta} v_2(\tau(t),\eta_1,\eta_2)$$

Have,

$$u_1(t,x_1,x_2) = Ae^{-\int_0^t k_1(\zeta)d\zeta} (c\tau(t)-\xi)_+^{\gamma_1},$$

$$u_2(t,x_1,x_2) = Be^{-\int_0^t k_2(\zeta)d\zeta} (c\tau(t)-\xi)_+^{\gamma_2}, \ c>0.$$

Due to the fact that

$$[b\tau(t) - \int_0^t l_i(\eta)d\eta - x_i] = 0,$$

If,

$$x_i \geq [b\tau(t) - \int_0^t l_i(\eta)d\eta - x_i] < 0, \ \forall t>0,$$

Then,

$$u_1(t,x) \equiv 0, \ u_2(t,x) \equiv 0, x_i \geq [b\tau(t) - \int_0^t l_i(\eta)d\eta - x_i] < 0, \ \forall t>0, \ i=1,2.$$

Therefore, the condition of localization of solutions of system (2) there are conditions

$$\int_0^{\mathring{a}} l_i(y)dy < 0,\ \tau(t) < \infty \text{ для } \forall t > 0,\ i = 1,2. \tag{10}$$

Condition (10) is the condition for the appearance of a new effect – the localization of the wave solutions (2). If the condition (10) is not executed, then there is the phenomenon of finite speed of propagation of disturbances, i.e.

$u_i(t,x) \equiv 0$ at $|x| \geq b(t)$, $\tau(t) = \int_0^t e - ^{(m_1+p-3)\int_0^{\zeta} k_1(y)dy} d\zeta$, moreover, the front goes arbitrarily far away, with increasing time, as $\tau(t) \to \infty$ at $t \to \infty$.

Investigation of qualitative properties of the system (2) is allowed to perform a numerical experiment depending on the values included in the numeric parameters. For this purpose, as the initial approximation was used to construct asymptotic solutions. The numerical solution of the problem for the linearization of the system (2) was used linearization methods of Newton and Picard. To build self-similar system of equations for biological populations, we used the method of nonlinear splitting [16, 19].

3. DEVELOPMENT OF EFFICIENT DIFFERENCE SCHEMES AND ALGORITHMS

In the domain $\overline{G} = \{(t,x): t \in [0,T],\quad x \in [a,b]\}$ considers the system of quasilinear equations of parabolic type

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial}{\partial x}\left(D_1 v^{m_1-1} \left|\dfrac{\partial u}{\partial x}\right|^{p-2} \dfrac{\partial u}{\partial x} \right) + \\ + l(t)\dfrac{\partial u}{\partial x} + k_1(t) u \left(1 - v^{\beta_1}\right), \\ \dfrac{\partial v}{\partial t} = \dfrac{\partial}{\partial x}\left(D_2 u^{m_2-1} \left|\dfrac{\partial v}{\partial x}\right|^{p-2} \dfrac{\partial v}{\partial x} \right) + \\ + l(t)\dfrac{\partial v}{\partial x} + k_2(t) v \left(1 - u^{\beta_2}\right), \end{cases} \tag{11}$$

With the initial

$$\begin{aligned} &u(0,x) = u_0(x) \geq 0, \quad x \in [a,b], \\ &v(0,x) = v_0(x) \geq 0, \quad x \in [a,b], \end{aligned} \tag{12}$$

And boundary conditions

$$\begin{aligned} &u(t,a) = \varphi_1(t) \geq 0, \quad t \in [0,T], \\ &u(t,b) = \varphi_2(t) \geq 0, \quad t \in [0,T], \\ &v(t,a) = \psi_1(t) \geq 0, \quad t \in [0,T], \\ &v(t,b) = \psi_2(t) \geq 0, \quad t \in [0,T]. \end{aligned} \tag{13}$$

Here $m_1,\ m_2,\ \beta_1,\ \beta_2,\ p$ - positive constants, $u_0(x)$ and $v_0(x)$ - initial distribution respectively for the first and second components, $\varphi_1(t)$ - value of the first components on the left border, $\varphi_2(t)$ - value of the first component on the right end, $\psi_1(t)$ and $\psi_2(t)$ - respectively, for second component.

Construct uniform grid in t and x $\overline{\omega_{\tau h}} = \{t_j = j\tau,\ j = 0,1,...,m, \tau m = T;\ x_i = a + ih, i = 0,1,...,n, h = \frac{b-a}{n}\}$ и approximate problem (11) - (13) by the balance method (integro-interpolation method)

$$\begin{cases} \dfrac{y_i^{j+1} - y_i^j}{\tau} = \dfrac{1}{h}\left(a_{i+1}\dfrac{y_{i+1}^{j+1} - y_i^{j+1}}{h} - a_i\dfrac{y_i^{j+1} - y_{i-1}^{j+1}}{h}\right) + l_i^{j+1}\dfrac{y_{i+1}^{j+1} - y_{i-1}^{j+1}}{2h} + k_{1i}^{j+1}y_i^{j+1}\left(1-\left(w_i^j\right)^{\beta_1}\right), \\ \dfrac{w_i^{j+1} - w_i^j}{\tau} = \dfrac{1}{h}\left(b_{i+1}\dfrac{w_{i+1}^{j+1} - w_i^{j+1}}{h} - b_i\dfrac{w_i^{j+1} - w_{i-1}^{j+1}}{h}\right) + l_i^{j+1}\dfrac{w_{i+1}^{j+1} - w_{i-1}^{j+1}}{2h} + k_{2i}^{j+1}w_i^{j+1}\left(1-\left(y_i^{j+1}\right)^{\beta_2}\right) \end{cases} \tag{14}$$

Where a_i and b_i calculated by methods

$$a_i(y) = 0.5D_1\left[\left(w_{i+1}^j\right)^{m_1-1}\left|\frac{y_{i+1}^{j+1} - y_i^{j+1}}{h}\right|^{p-2} + \left(w_i^j\right)^{m_1-1}\left|\frac{y_i^{j+1} - y_{i-1}^{j+1}}{h}\right|^{p-2}\right]$$

And,

$$b_i(w) = 0.5D_2\left[\left(y_{i+1}^{j+1}\right)^{m_2-1}\left|\frac{w_{i+1}^{j+1} - w_i^{j+1}}{h}\right|^{p-2} + \left(y_i^{j+1}\right)^{m_2-1}\left|\frac{w_i^{j+1} - w_{i-1}^{j+1}}{h}\right|^{p-2}\right]$$

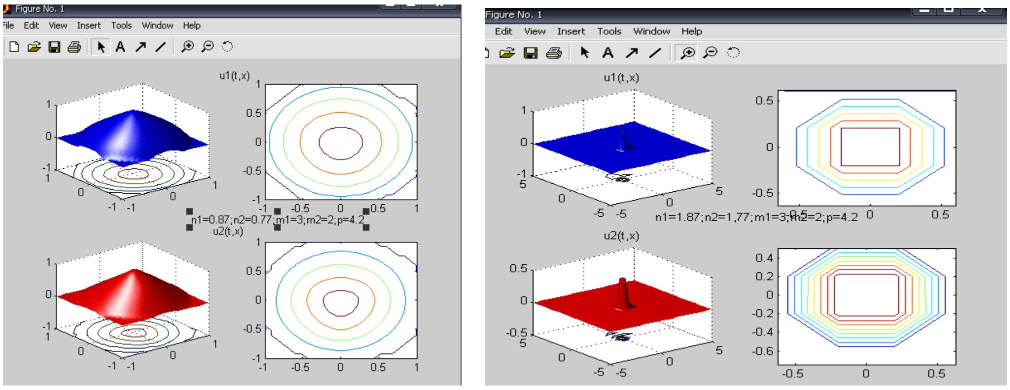
The scheme system (17) is nonlinear, with respect to the function y^{j+1} and w^{j+1}, for finding its solution using a method of iterations. The iterative process builds as follows

$$\begin{cases} \dfrac{\overset{s+1}{y}{}_i^{j+1} - y_i^j}{\tau} = \dfrac{1}{h}\left(\overset{s}{a}_{i+1}\dfrac{\overset{s+1}{y}{}_{i+1}^{j+1} - \overset{s+1}{y}{}_i^{j+1}}{h} - \overset{s}{a}_i\dfrac{\overset{s+1}{y}{}_i^{j+1} - \overset{s+1}{y}{}_{i-1}^{j+1}}{h}\right) + l_i^{j+1}\dfrac{\overset{s+1}{y}{}_{i+1}^{j+1} - \overset{s+1}{y}{}_{i-1}^{j+1}}{2h} + \\ +k_{1i}^{j+1}\overset{s+1}{y}{}_i^{j+1}\left(1-\left(w_i^j\right)^{\beta_1}\right) \\ \dfrac{\overset{s+1}{w}{}_i^{j+1} - w_i^j}{\tau} = \dfrac{1}{h}\left(\overset{s}{b}_{i+1}\dfrac{\overset{s+1}{w}{}_{i+1}^{j+1} - \overset{s+1}{w}{}_i^{j+1}}{h} - \overset{s}{b}_i\dfrac{\overset{s+1}{w}{}_i^{j+1} - \overset{s+1}{w}{}_{i-1}^{j+1}}{h}\right) + l_i^{j+1}\dfrac{\overset{s+1}{w}{}_{i+1}^{j+1} - \overset{s+1}{w}{}_{i-1}^{j+1}}{2h} + \\ +k_{2i}^{j+1}\overset{s+1}{w}{}_i^{j+1}\left(1-\left(y_i^{j+1}\right)^{\beta_2}\right) \end{cases} \tag{15}$$

Regarding the function $\overset{(s+1)}{y}{}^{j+1}$ and $\overset{(s+1)}{w}{}^{j+1}$ difference scheme (15) becomes linear. As the initial iteration are the functions y and w of previous time step: $\overset{(0)}{y}{}^{j+1} = y^j$ and $\overset{(0)}{w}{}^{j+1} = w^j$. For convergence of the iteration, we require the conditions

$$\max_i \left| \overset{(s+1)}{y_i} - \overset{(s)}{y_i} \right| \leq \varepsilon \text{ and } \max_i \left| \overset{(s+1)}{w_i} - \overset{(s)}{w_i} \right| \leq \varepsilon .$$

For the solution of linear circuits (15), with conditions (12)-(13) on the grid, use the sweep method. The following are the results of numerical experiments for different values of parameters (Fig. 1).



$x_1 = 1; x_2 = 1$ $x_1 = 2; x_2 = 2$

Figure 1: Results of Computational Experiment

4. CONCLUSIONS

Thus, the proposed nonlinear mathematical model of biological populations with double-linearity properly describes the studied process. A numerical study of nonlinear processes described by equations with a double nonlinearity and analysis of the results, based on the obtained estimates of the solutions gives a comprehensive picture of the process in two-component systems competing biological population with the preservation of localization properties in the target area and the size of the outbreak. The results will further provide an opportunity to assess the speed of propagation of diffusive waves.

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